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Cosmic Microwave Background:

The universe is not an exact FRW universe but rather a perturbed FRW one. Otherwise, if it was exactly homogenous and isotropic, the structure would never form. The problem of structure formation is one of the most important problems in Cosmology.

The CMB observations tell us that the inhomogeneitis were initially very small $\sim O(10^{-5})$. The question then is how these tiny inhomogeneitis evolved to form structures of various sizes (galaxies, clusters, ...). The temperature anisotropy in the CMB gives information about the primordial inhomogeneities. One can find the two-point correlation function of temperature anisotropy $\frac{\delta T}{T}$ between points on the surface of last scattering. Expanding this in terms of spherical harmonics, one can then find the CMB power spectrum. The multipoles are related to the angular distance θ through $\ell = \frac{180^\circ}{\theta}$ ($\ell \geq 1$). The power spectrum at small ℓ ($< O(10)$) gives information about the size of primordial inhomogeneities. This includes:

(1) $\frac{\delta\rho}{\rho} \sim O(10^{-5})$.

(2) $\frac{\delta\rho}{\rho}(\ell)$ is nearly scale invariant at small ℓ.

The first piece of information is an input from observation. However, the second one is a curious fact as inhomogeneities with small ℓ had a wavelength larger than the horizon size at the time of recombination. The question arises then that how modes that were not in causal contact had almost the same amplitude.

Some possible scenarios for the origin of density perturbation include thermal fluctuations and statistical fluctuations. It turns out that thermal fluctuations cannot provide a satisfactory explanation since they have subhorizon size at the time of generation and also damp down due to Hubble expansion. As it turns out, statistical fluctuation cannot successfully explain the origin of CMB anisotropy either. One can show that Poisson fluctuations give rise to too large inhomogeneities. Currently, inflation is the dominant paradigm for generating tiny perturbations. The simplest models of inflation predict a nearly scale-invariant spectrum with Gaussian statistics, which is in agreement with the CMB data. We do not wish to discuss inflation and how it generates density perturbations in more detail at this point.

Once we have a scenario for generating perturbations, the next question is their evolution. One can divide this into two different regimes:

(1) Super horizon perturbations. In this regime the problem is how a three-dimensional hypersurface evolves in time. For

a perfect FRW universe, a homogeneous and isotropic hypersurface will always remain homogeneous and isotropic. The situation is different for a perturbed FRW universe. The four-dimensional spacetime can be sliced in many ways, which implies various choices for the three-dimensional hypersurfaces. These hypersurfaces have both density perturbations and curvature perturbations. However, the fact that we see a temperature anisotropy of $\sim O(10^{-5})$ in the CMB, is independent of the choice of hypersurfaces. Thus, like any gauge theory, we have to find gauge-invariant parameters in order to compare observables with theory predictions. We note that general relativity is a gauge theory due to its invariance under general coordinate transformations.

We do not discuss this in any further detail here, and just point out that it is a very non-trivial issue that requires a careful treatment within general relativity.

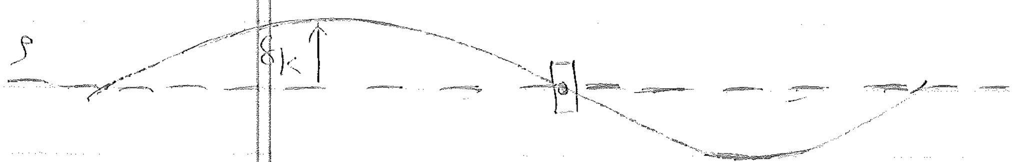
(2) Subhorizon perturbations. Modes that have a wavelength smaller than the horizon size evolve based on causal physics. It is important to note that in a radiation-dominated or matter-dominated universe a superhorizon mode eventually becomes a subhorizon mode. This can be seen a $\lambda_{phys} \propto a^{-1}$ ($\propto t^{\frac{1}{2}}$ for radiation domination and $\propto t^{\frac{2}{3}}$ for matter domination), while the horizon size $\sim H^{-1} \propto t$. Therefore, the horizon will catch up with the wavelength at some point in time.

Evolution of subhorizon perturbations is essentially described by fluid dynamics as one has a perturbed perfect fluid. Let us start by a qualitative discussion and consider a single inhomogeneous mode with wavenumber "k" (ignoring expansion of the universe for the time being):

$$\delta\rho = \delta_k e^{i\vec{k}\cdot\vec{r}} + \delta_k^* e^{-i\vec{k}\cdot\vec{r}} \qquad (\delta\rho \ll \rho)$$

There are two forces acting on the perturbations: (1) Force pressure F_p; and, (2) Force from gravity F_g.

Consider a tiny element of the fluid as follows:



After using the equation of state of the fluid $P = w\rho$, we find:

$$\vec{F_p} = -|\vec{\nabla} P| \, dV \sim k w \, \delta_k \, dV$$

Also:

$$\vec{F_g} \sim \frac{G \delta_k \rho}{k} \, dV$$

The important point is the sign difference between $\vec{F_p}$ and $\vec{F_g}$. Pressure wants to smooth out the perturbations in a fluid, while gravity wants to enhance it because of its attractive nature. Therefore, there exist two different regimes:

(a) $k < k_J$: $k_J \sim \frac{(G\rho)^{1/2}}{v_s}$ is the Jeans wavenumber, where $v_s = w^{\frac{1}{2}}$ is the speed of sound in the fluid. In thi case gravity wins over pressure and perturbations will grow.

(b) $k > k_J$: In this case pressure wins over gravity, and hence perturbations will oscillate.

We see a characteristic length $\lambda_J \sim \frac{v_s}{(G\rho)^{1/2}}$, called the "Jeans wavelength", that separates the stable (oscillatory) and unstable (growing) regimes of perturbations. If the perturbations have a sufficiently large wavelength $\lambda \gg \lambda_J$, then gravity wins and perturbation is unstable. If $\lambda < \lambda_J$, then pressure wins and perturbation is stable.

We will discuss this in a more precise and quantitative way that also includes expansion of the universe. We will see how in an expanding universe subhorizon perturbations evolve in the two regimes and make a transition from one regim to another one.